1) Find orbits and cycles:

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

$Orb(1) = Orb(2) = Orb(3) = Orb(4) = Orb(5) = \{1,2,3,4,5\}$
$Orb(6) = \{6\}$
$Orb(7) = \{7\}$
$Orb(8) = Orb(9) = \{8,9\}$

<u>cycles:</u> $(1,2,3,4,5)$; $(8,9)$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$

$Orb(1) = Orb(6) = Orb(2) = Orb(5) = \{1,6,2,5\}$
$Orb(3) = Orb(4) = \{3,4\}$

cycles: $(1,6,2,5)$; $(3,4)$

2) (a) $(1,2,3,4,5,6,7,8) = (1,2,3,4,5) \circ (8,9)$

(b) $(1,2,3,4,5,6) = (1,6,2,5) \circ (3,4)$

3) (a) $(1,2,3)(4,5)(1,6,7,8,9)(1,5)$

$= \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} 4 & 5 \\ \downarrow & \downarrow \\ 5 & 4 \end{pmatrix} \circ (1,6,7,8,9)}$

$\underbrace{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 3 & 6 & 5 & 4 & 2 & 8 & 9 & 1 \end{pmatrix}}_{} \circ (1,5)$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 3 & 6 & 1 & 4 & 2 & 8 & 9 & 5 \end{pmatrix}$

4) We are going to do a proof by induction.

Base case: $n=2$ $(1 \to 2)$

$$\begin{pmatrix} 1 & 2 \\ \downarrow & \downarrow \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ \downarrow & \downarrow \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ \downarrow & \downarrow \\ 1 & 2 \end{pmatrix} = Id$$

the Base case is good

Hérédité:

Hyp: $P(n)$: $(1 \to \ldots \to n)^{-1} = (n \to n-1 \to \ldots \to 2 \to 1)$

$P(n+1)$: $\left[(1 \to \ldots \to n) \circ (1 \to n+1)\right]^{-1} =$

$= (1, \ldots, n+1)$

$(1 \to n+1) \circ (n, \ldots, 1)$
$= (n+1, n, \ldots, 1)$

□