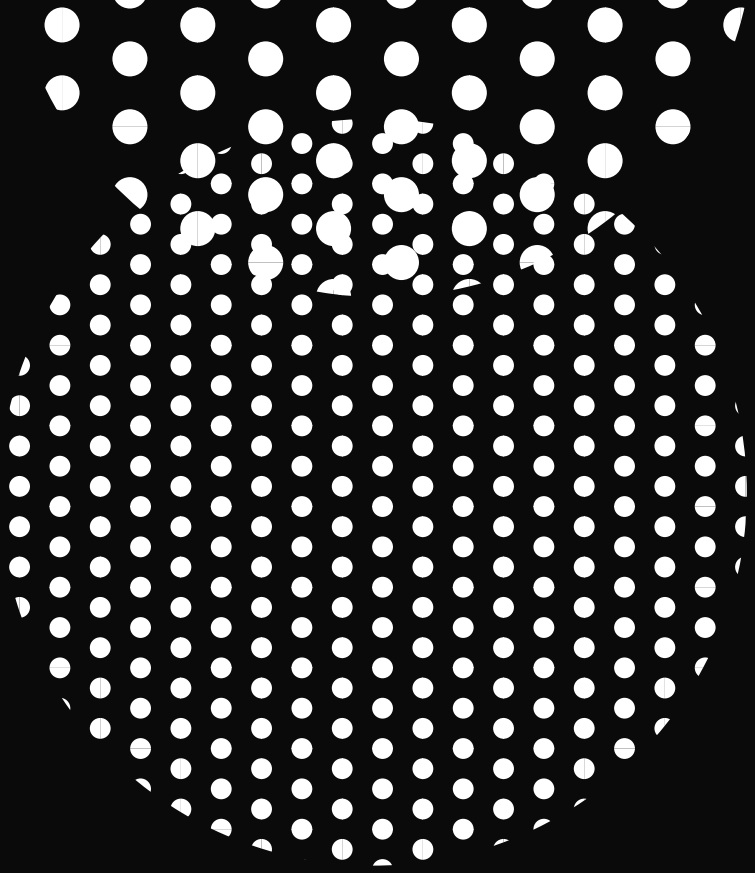

Exercice 1

$a = 4 - (1 - i) = 3 + i$

$b = (2+i)(1-4i) = 2 + i - 8i + 4 = 6 - 7i$

$c = \frac{1}{i} = \frac{-i}{1} = -i$

$d = \frac{-2}{1 - i\sqrt{3}} = -2 \frac{1}{1 - i\sqrt{3}} = -2 \frac{1 + i\sqrt{3}}{4} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$e = \frac{3+6i}{3-4i} = \frac{(3+6i)(3+4i)}{9+16} = \frac{9 - 24 + 18i + 12i}{25} = \frac{-15 + 30i}{25} = \frac{3+6i}{5}$
$= \frac{3}{5} + \frac{6}{5}i$

Exercice 3

$2 + 5\bar{z} = 7 - 8i$

$\Leftrightarrow a + bi + 5a - 5bi = 7 - 8i$
$\Leftrightarrow 6a - 4bi = 7 - 8i$
$\Leftrightarrow a = \frac{7}{6} \quad b = 2$

$\Rightarrow z = \frac{7}{6} + 2i$

Exercice 6

$|1 - i| = \sqrt{1+1} = \sqrt{2}$

$|(1-i)(3+4i)| = |3 - 3i + 4i + 4| = |7 - i| = \sqrt{49+1} = \sqrt{50}$

Exercice 19

$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{100} = \left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)^{100} = \cos\left(\frac{96\pi}{3} + \frac{4\pi}{3}\right) + i\sin\left(\frac{96\pi}{3} + \frac{4\pi}{3}\right)$

$= \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$

$= -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Exercice 21

$$0+i = 1e^{i\frac{\pi}{2}}$$

$$\sqrt{i} = \sqrt{0+i} = \sqrt{e^{i\frac{\pi}{2}}} = e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

Exercice 22

$$z^2+z+1=0$$

$$\Delta = 1-4 = -3 \qquad \sqrt{-3} = i\sqrt{3}$$

$$z_1 = \frac{-1+i\sqrt{3}}{2}$$

$$z_2 = \frac{-1-i\sqrt{3}}{2}$$