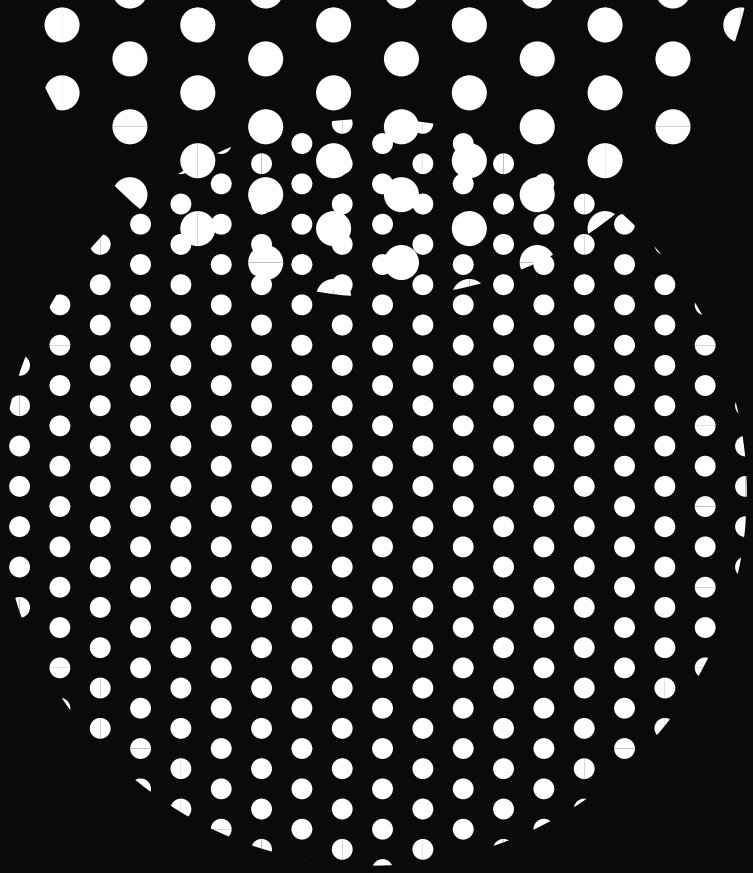

Exercice 1

a) $(a\ b)\circ(c\ d)\circ(d\ a)$

$c \to c \to d \to d$

$d \to a \to a \to b$

$a \to d \to c \to c$

$b \to b \to b \to a$

$=$

$(a\ c\ d\ b)$

b) $\underset{\sigma_1 = (1\,2)(3\,4)}{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}} \circ \underset{\sigma_2 = (1\ 4\ 3\ 5)}{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}}$ donc $\sigma_1 \circ \sigma_2 = (1\ 3\ 5\ 2)$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 4 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 5 & 4 & 3 & 2 & 6 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \overset{\varepsilon(-1)}{=} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 6 & 1 \end{pmatrix} = (1\ 5\ 6)\circ(2\ 4)$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 1 & 5 \end{pmatrix} = (1\ 6\ 5)\circ(2\ 4)$

Exercice 2

$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 6 & 2 & 1 \end{pmatrix} = \underset{\sigma_{1,1}}{(1\ 3\ 4\ 6)} \circ \underset{\sigma_{1,2}}{(2\ 5)}$

$o(\sigma_1) = ppcm(o(\sigma_{1,1}), o(\sigma_{1,2})) = ppcm(4, 2) = 4$ $\quad \sigma^{100} = (\sigma^4)^{25} = Id$

$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 1 & 2 & 4 \end{pmatrix} = \underset{\sigma_{2,1}}{(1\ 3\ 6\ 2\ 5)} \circ \underset{\sigma_{2,2}}{(4\ 7)}$

$o(\sigma_2) = ppcm(o(\sigma_{2,1}), o(\sigma_{2,2})) = ppcm(5, 2) = 10$ $\quad \sigma_2^{100} = (\sigma_2^{10})^{10} = Id$

$\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 9 & 7 & 2 & 5 & 8 & 1 & 3 \end{pmatrix} = \underset{\sigma_{3,1}}{(1\ 4\ 7\ 8)} \circ \underset{\sigma_{3,2}}{(2\ 6\ 5)} \circ \underset{\sigma_{3,3}}{(3\ 9)}$

$\sigma_{3,2}^4 = \sigma_{3,2}^2$

$o(\sigma_3) = ppcm(4, 3, 2) = 12$

$\sigma_3^{100} = \sigma_3^{96} \circ \sigma^4 = \sigma^4 = (2\ 6\ 5)$

$\mathfrak{S}_n = S_n$ ou $\mathcal{S}_n$ gpe symmetrique

<u>Exercice 3</u> $n \geq 2$ $2 \leq k \leq n$

$|S_n| = n!$

$A_n^k = \#[k \text{ cycles de } \mathfrak{S}_n] = \frac{n(n-1)(n-2)\dots(n-k+1)}{k}$

$\frac{A_n^k}{k} = \frac{n!}{(n-k)!\,k}$

<u>Exercice 4</u>

$\sigma \in S_8$ $\exists r > 0$ tq

$\sigma = \sigma_1 \dots \sigma_r$ à supp disjoint

donc $r \leq 8$

σ_i k-cycle $1 \leq k \leq 8$

$o(\sigma) \mid \text{ppcm}(1, \dots, 8) = 2^3 \cdot 3 \cdot 5 \cdot 7$

si $8 \mid o(\sigma)$

σ - 8 cycle

si $4 \mid o(\sigma)$ mais $8 \nmid o(\sigma)$

$\sigma = (4 \text{ cycle})()$ $\begin{cases} (2 \text{ cycle})(2 \text{ cycle}) & \to o(\sigma) = 4 \\ (3 \text{ cycle})() & \to o(\sigma) = 12 \\ (4 \text{ cycle}) & \to o(\sigma) = 4 \\ ()()()() & \to o(\sigma) = 4 \end{cases}$

si $7 \mid o(\sigma)$

$\sigma = (7 \text{ cycle})()$ $o(\sigma) = 7$

si $5 \mid o(\sigma)$

$\sigma = (5 \text{ cycle})() \begin{cases} (3 \text{ cycle}) & o(\sigma) = 15 \\ (2)() & o(\sigma) = 10 \\ ()()() & o(\sigma) = 5 \end{cases}$

si $3 \mid o(\sigma)$ et $4 \nmid o(\sigma)$
$5 \nmid o(\sigma)$
$7 \nmid o(\sigma)$

$\sigma =$ (3 cycle) $\begin{cases} (2) \quad (2) \\ ()()()()() \\ (3)\ (2) \end{cases}$ | $o(\sigma) = 6$
$o(\sigma) = 3$
$o(\sigma) = 6$

$(2)\ ()()()$ | $o(\sigma) = 6$
$(3)\ ()()$ | $o(\sigma) = 3$

Sinon $o(\sigma) = 2$

$\sigma =$ (2 cycle) $()()()()()()$
$(2\quad)(2\quad)()()()()$
$(2\quad)(2\quad)(2\quad)()$
$(2\quad)(2\quad)(2\quad)(2)$ $\quad o(\sigma) = 2$

resultat : 15

Exercice 5

Signature
$\sigma = \tau_1 \circ .. \circ \tau_q$ où τ_i transposition

$\varepsilon(\sigma) = (-1)^q$

Exercice 6

$A_n = \{\sigma \in \mathfrak{S} \,/\, \varepsilon(\sigma) = 1\}$ $\qquad A_4 = \{(1), (3), (4)\}$

$\mathfrak{S}_4 = \{ \underset{(1)}{id}, \underset{i \neq j}{(i\ j)}_{(2)}, \underset{i \neq j \neq k}{(i\ j\ k)}_{(3)}, \underset{(4)}{(i\ j)(k\ l)}, \underset{(5)}{(i\ j\ k\ l)} \}$

(1) $Id = (1\ 2)(1\ 2)$ $\qquad \varepsilon(Id) = 1$
(2) $(i\ j)$ $\qquad \varepsilon((i\ j)) = -1$
(3) $(i\ j\ k) = (i\ j)(j\ k)$ $\qquad \varepsilon(3) = 1$
(4) $(i\ j)(k\ l)$ $\qquad \varepsilon(4) = 1$

(5) $(i\ j\ k\ l) = (i\ j)(j\ k)(k\ l)$ $\qquad \varepsilon(5) = -1$

$\# A_4 = 12$

B) $H = \{ Id, (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3) \}$

$$A_4 = \left\{ \begin{array}{lll} Id, & (1\,2\,3), (1\,3\,2) & , (1\,2)(3\,4), \\ & (1\,3\,4), (1\,4\,3) & , (1\,3)(2\,4), \\ & (1\,2\,4), (1\,4\,2) & , (1\,4)(2\,3) \\ & (2\,3\,4), (2\,4\,3) & , \end{array} \right\}$$

$\# H = 4 \mid \# A_4 = 12$

$\sigma \in H, \quad \sigma^{-1} = \sigma \in H \qquad Id \in H$

$\sigma \circ \sigma' \in H, \qquad \sigma, \sigma' \in H$

d'où H est un ss-gpe de A_4