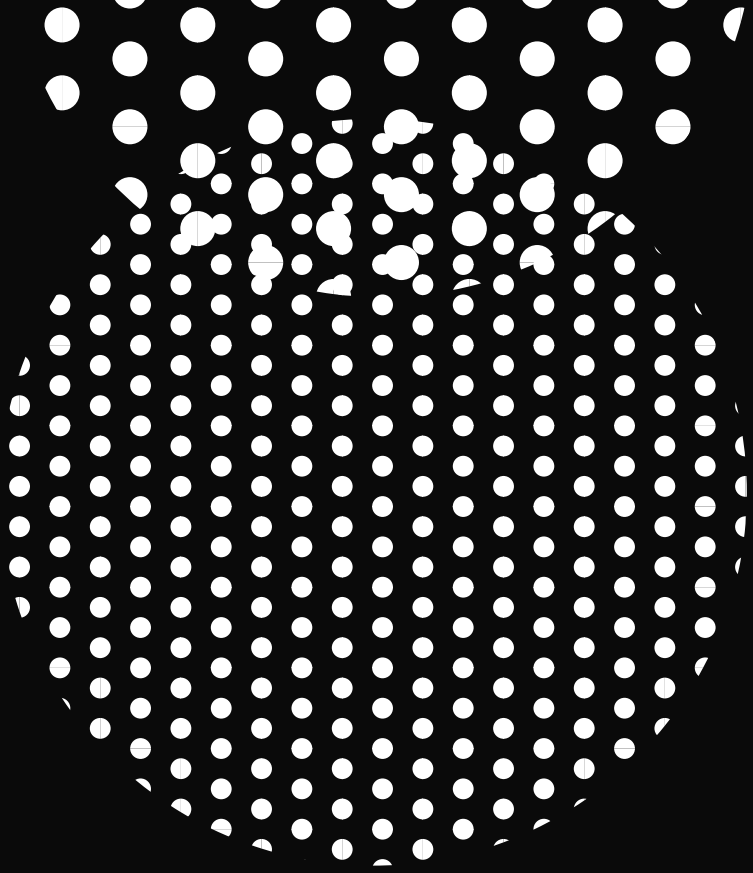

Exercices

$n \geq 2 \qquad x \in (\mathbb{Z}/n\mathbb{Z})^*$

a) $n = 10$

$\#(\mathbb{Z}/10\mathbb{Z})^* = \varphi(10) = \varphi(2)\varphi(5) = (2-1)(5-1) = 4$

$(\mathbb{Z}/10\mathbb{Z})^* = \{\bar{1}, \bar{3}, \bar{7}, \bar{9}\}$

$o(1) = 1 \quad \bar{3}, \bar{3}^2 = 9 \quad$ donc $\quad o(\bar{3}) = 4$

$\bar{7}, \overline{49} = \bar{9} \quad$ donc $\quad o(\bar{7}) = 4$

$\bar{9}, \bar{9}^2 = 1 \quad$ donc $\quad o(\bar{9}) = 2$

$\cdot n = 8$

$\#(\mathbb{Z}/8\mathbb{Z})^* = \varphi(8) = \varphi(2^3) = 2^2(2-1) = 4$

$(\mathbb{Z}/8\mathbb{Z})^* = \{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$

$o(1) = 1$

$\bar{3}, \bar{3}^2 = \bar{9} = \bar{1} \quad o(3) = 2$

$\bar{5}, \bar{5}^2 = \overline{25} = \bar{1} \quad o(5) = 2$

$\bar{7}, \bar{7}^2 = \overline{49} = \bar{1} \quad o(7) = 2$

ss-gpes $(\mathbb{Z}/10\mathbb{Z})^*$: $\{1\}, \{1, 9\}, \{1, 3, 7, 9\}$

ss-gpes $(\mathbb{Z}/8\mathbb{Z})^*$: $\{1\}, \{1, 3\}$
$\{1, 5\}, \{1, 7\}, \{1, 3, 5, 7\}$

b) $n = 7 \qquad \#(\mathbb{Z}/7\mathbb{Z})^* = 6$

$o(\bar{1}) = 1$

$\bar{2}, \bar{2}^2 = 4, \bar{2}^3 = \bar{8} = \bar{1}$ donc $o(\bar{2}) = 3$

$\bar{3}, \bar{3}^2 = \bar{2}, \bar{3}^3 = \bar{6} = \bar{-1}$ donc $o(\bar{3}) = 6$

$\bar{4}, \bar{4}^2 = \bar{2}, \bar{4}^3 = 1$ donc $o(\bar{4}) = 3$

$\bar{5}, \bar{5}^2 = \overline{25} = \bar{4}, \bar{5}^3 = \overline{20} = \bar{6}$ donc $o(\bar{5}) = 6$

$\bar{6}, \bar{6}^2 = \overline{36} = \bar{1}$ donc $o(\bar{6}) = 2$

donc $\forall a \in (\mathbb{Z}/7\mathbb{Z})^* \quad \bar{a}^6 = 1$

c) $n = 10$

$3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$ $3^4 = 81$

chiffres les unités de $3^k = \begin{cases} 1 & \text{si } k \equiv 0\ [4] \\ 3 & \text{si } k \equiv 1\ [4] \\ 9 & \text{si } k \equiv 2\ [4] \\ 7 & \text{si } k \equiv 3\ [4] \end{cases}$

et $o(\bar{3}) = 4$

d) $n \geq 2$

$o(\bar{1}) = 1$

$o(-\bar{1}) = \begin{cases} 2 & \text{si } n > 2 \\ 1 & \text{si } n = 2 \end{cases}$