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Exercice 1

1)

20 per inch^2 per minute

5 inch^2 in t mins.

$$P(X=k) = \frac{\lambda^k}{k} e^{-\lambda} \quad \text{we choose } \lambda = 20 \cdot 5 \cdot t = \frac{300}{60} = 5$$

$$t = \frac{3}{60}$$

then $P(X=0) = \frac{5^0}{0!} e^{-5} = e^{-5}$

2) N = "num of students that have the same seat for Both classes"

E_j = "j^{th} student has the same seat for Both classes"

$$P(N=0) = 1 - P\left(\bigcup_{j=1}^{100} E_j\right)$$

Exercice 2

100 places

H_i = "i^{th} student has the same seat for Both courses"

X - number of students that have the same paces

a)

$$P(H_i) = \frac{99}{100}$$

$$P(X=0) = 1 - P\left(\bigcup_{i=1}^{100} H_i\right) =$$

$$P\left(\bigcup_{i=1}^{100} H_i\right) = \sum_{i=1}^{100} (-1)^{j-1} \binom{100}{i} P\left(\bigcap_{k=1}^{i} H_k\right)$$

$(-1)^{j-1}$: Because inclusion-exclusion alternates

$\binom{100}{i}$: num. of ways to take exactly i elems

$P\left(\bigcap_{k=1}^{i} H_k\right)$: intersection of those elems $= \frac{(100-i)!}{100!}$

cause there are 100! ways to choose seats for all 100 students and we fix i places for i students. then for the rest 100-i students there are (100-i)! ways to sit.

$$= \sum_{i=1}^{100} (-1)^{i-1} \frac{100!}{i!(100-i)!} \cdot \frac{(100-i)!}{100!} = \sum_{i=1}^{100} \frac{(-1)^{i-1}}{i!}$$

$$P(N=0) = 1 - \sum_{i=1}^{100} \frac{(-1)^{i-1}}{i!} = \sum_{i=1}^{100} \frac{(-1)^i}{i!}$$

b) $\sum_{i=1}^{100} \frac{(-1)^i}{i!} \approx e^{-1}$

$$N = \sum_{i=1}^{100} I_i$$

In addition: we define I_i an event that a student has the same seat for two lectures $P(I_i) = \frac{1}{100}$

$P(I_i \cap I_j) = \frac{1}{100} \cdot \frac{1}{99} \approx \frac{1}{100^2} = P(I_i) P(I_j)$ (weakly dependent)

thus it's approximately $N \sim Pois(\lambda)$ with $\lambda = E[N] = 100\, E[I_i] = 1$

thus $P(N=0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda} = e^{-1}$

c) $P(N \geq 2) = 1 - P(N=1) - P(N=2) = 1 - e^{-1} - e^{-1} = 1 - 2e^{-1}$

<u>Exercice 3</u>

a) $X \sim Pois(\lambda)$ $\qquad \theta = e^{-3\lambda}$

$T = e^{-3X}$

$$E[T] = \sum_{k=1}^{100} e^{-3k} P(X=k) = \sum_{k=1}^{100} e^{-3k} \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \sum_{k=1}^{100} \frac{(e^{-3}\lambda)^k}{k!}$$

$$= e^{-\lambda} e^{e^{-3}\lambda} = e^{\lambda(e^{-3}-1)}$$

$E[T] - \theta = e^{\lambda(e^{-3}-1)} - e^{-3\lambda} \neq 0$ so it's biased

b) $g(X) = (-2)^X$

$$E[g(X)] = \sum_{k=0}^{100} (-2)^k \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \sum_{k=0}^{100} \frac{(-2\lambda)^k}{k!} = e^{-\lambda} e^{-2\lambda} = e^{-3\lambda}$$

$E[g(X)] - \theta = e^{-3\lambda} - e^{-3\lambda} = 0$ thus $g(X)$ is an unbiased estimator.