Discrete	Continuous
PMF $P(X=x)$	PDF $f_X(x) = F_X'(x)$ $[P(X=x)=0]$ expl. ↓
CDF $F_X(x) = P(X \le x)$	CDF $F_X(x) = P(X \le x)$
$E(X) = \sum_{x \in X(\Omega)} x P(X=x)$	$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx$
$Var(X) = E[X^2] - E[X]^2$	$Var(X) = E[X^2] - E[X]^2$
LOTUS $E[g(X)] = \sum_x g(x) P(X=x)$	LOTUS $E[g(X)] = \int g(x) f_X(x)\,dx$

PDF = probability <u>density</u> function ⇐ $P(a \le X \le b)$

↳ густина

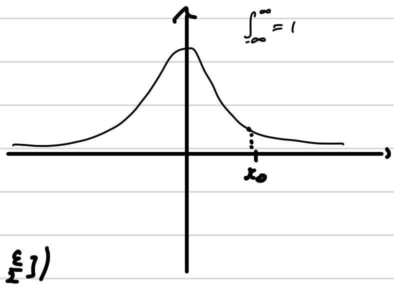
<u>Def:</u> r.v X has PDF $f(x)$ if $P(a \le X \le b) = \int_a^b f(x)\,dx$ $\forall a,b$

if $a = b \Rightarrow P(a \le X \le a) = P(X=a) = \int_a^a f(x)\,dx = 0$

To be valid:

→ $f(x) \ge 0$

→ $\int_{-\infty}^{\infty} f(x) = 1$



$f(x_0)$ $\varepsilon > 0$ (small)

$f(x_0) \cdot \varepsilon \approx P(X \in [x_0 - \frac{\varepsilon}{2}, x_0 + \frac{\varepsilon}{2}])$

If X has PDF, CDF is $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$

If X has CDF $F(x)$ C^0, then $f(x) = F'(x)$

$P(a < X < b) = \int_a^b f(x)dx = F(a) - F(b)$

<u>Variants</u>

how far the value from its mean ($E[X]$)

$$Var(X) = E[X - E[X]] \overset{\text{linearity}}{=} E[X] - E[\underbrace{E[X]}_{\text{const}}] = E[X] - E[X] = 0$$

problem, another def:

$$Var(X) = E[(X - E[X])^2]$$

on average how far X from its mean (squared)

Standart deviaton : $SD(X) = \sigma(X) = \sqrt{Var(X)}$
(Ecart type)

<u>Another way to express variance:</u>

$$Var(X) = E(X^2 - 2XE[X] + E[X]^2) =$$

$$= E[X^2] - 2E[X]E[X] + E[X]^2 =$$

$$= E[X^2] - E[X]^2$$

<u>Distribution</u>

<u>Unif(a,b)</u>

completely "random point in $[a,b]$"

unif: prob. proportional to length

$$f(x) = \begin{cases} c, & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \Rightarrow 1 = \int_a^b c\,dx = c = \frac{1}{b-a}$$

$$F(x) = \int_{-\infty}^{x} f(t)dt \overset{x>a}{=} \int_a^x f(t)dt = \begin{cases} 0 \text{ if } x < a \\ \frac{x-a}{x-b}, \text{ if } a \le x < b \\ 1 \text{ if } x > b \end{cases}$$

$$E[X] = \int_a^b \frac{x}{b-a}\,dx = \frac{x^2}{2(b-a)}\Big|_a^b = \frac{1}{2(b-a)}(b-a)(b+a) = \frac{a+b}{2}$$ (average is in the middle)

$$E[X^2] = E[Y] \quad (Y = X^2) \qquad \text{(need PDF of } Y\text{)}$$

$$= \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$ (Law of unconscious statistician (LOTUS))

LOTUS

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$$

Let $U \sim \text{Unif}(0,1)$, $E(U) = \frac{1}{2}$, $E(U^2) = \int_0^1 u^2 \underbrace{f_U(u)}_{1}\,du = \frac{1}{3}$

$$\text{Var}(U) = E[U^2] - (E[U])^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

Uniform is universal

Let $U \sim \text{Unif}(0,1)$, F be a CDF (assume is strictly increasing continuous)

Thm: Let $X = F^{-1}(U)$, then $X \sim F$

proof: $P(X \leq x) = P(F^{-1}(U) \leq x) = P(U \leq F(x)) = F(x)$