



Def: $E[Y|X]$ is a conditional expectation.
Let A be an event and X a r.v then
$$E[X|A] = \sum_{x \in \mathcal{X}} x P(X=x|A)$$

Thm: Let A_1, \dots, A_n be a partition of the sample space
and Y a r.v then

$$E[Y] = \sum_{i=1}^n E[Y|A_i] P(A_i)$$

$$E[Y|X=x] = \sum_{y \in \mathcal{Y}} y P(Y=y|X=x)$$

$= g(x)$

Important: $E[E[Y|X]] = E[Y]$

$E[Y|X]$ is a r.v as well.