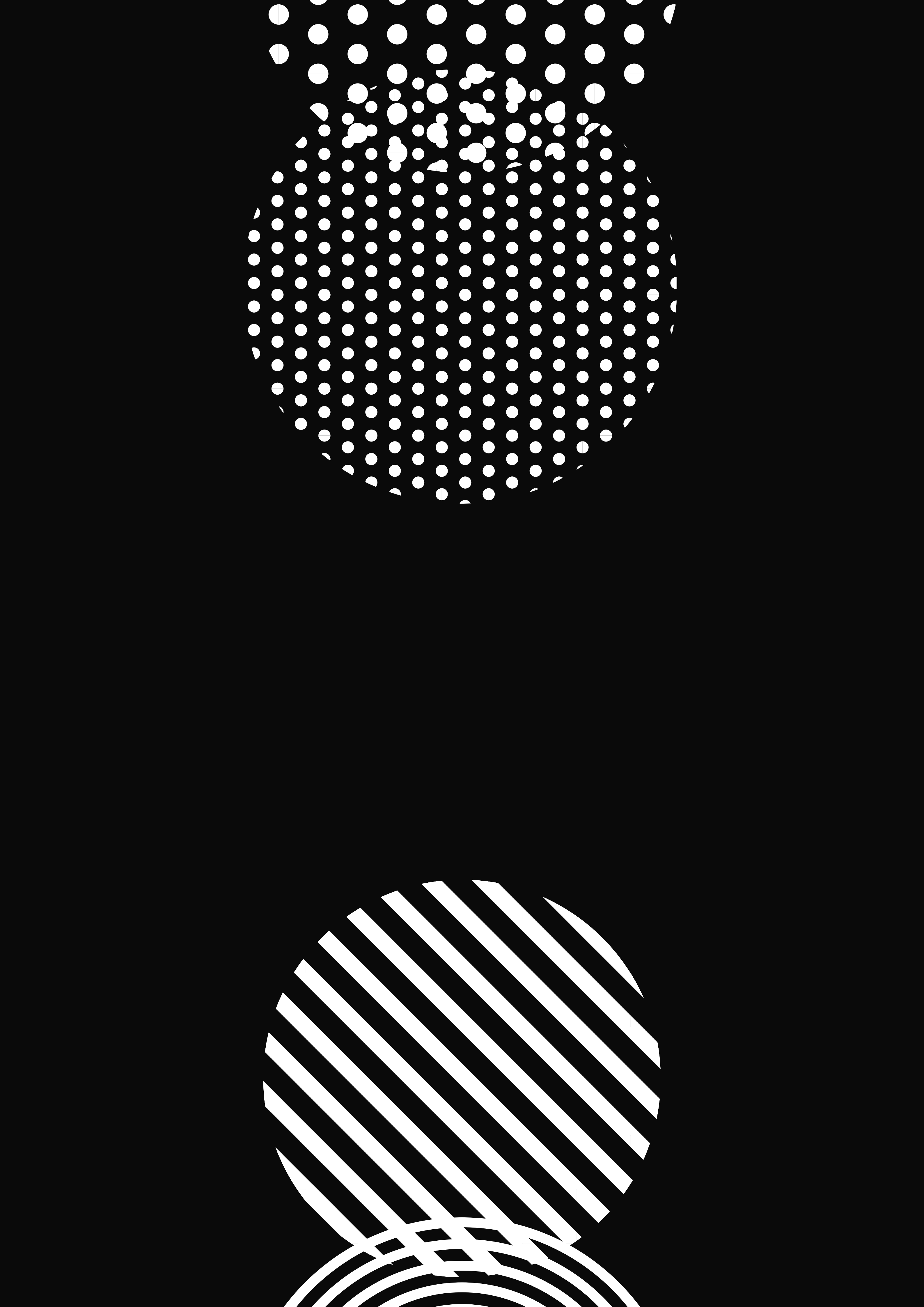

Exercice 4

$$f(x) = \int_0^x e^{-t^2}\, dt$$

$$g(x) = \int_0^1 \frac{e^{-(1+t^2)x^2}}{1+t^2}\, dt$$

1) $f'(x) = e^{-x^2}$

$$g'(x) = \int_0^1 -2x e^{-(1+t^2)x^2}\, dt = -2x e^{-x^2} \int_0^1 e^{-(tx)^2}\, dt$$

$$= -2\ e^{-x^2} \int_0^x e^{-v^2}\, dv \qquad dv = (xt)'\, dt$$

$$dv = x\,dt \Rightarrow dt = \frac{dv}{x}$$

$$=$$

$$h' = g' + (f^2)' = 2f \cdot f' + g' = 2e^{-x^2} \int_0^x e^{-t^2}\, dt + -2\ e^{-x^2} \int_0^x e^{-v^2}\, dt$$
$$= 0.$$

Ce qui signifie que h est constant.

2) $h(0) = \underset{0}{\underset{\shortparallel}{f(0)}}^2 + g(0) = g(0) = \int_0^1 \frac{1}{1+t^2}\, dt = [\arctan(t)]_0^1 = \frac{\pi}{4}$

d'où $h(0) = \frac{\pi}{4}$

3

Exercice 6

$$f(t) = \begin{cases} \dfrac{\sin(t)}{t} & \text{si } t \neq 0 \\ 1 & \text{si } t = 0 \end{cases}$$

continue et C^∞ sur $\mathbb{R}$

$$F(x) = \int_0^{+\infty} \underbrace{f(t)e^{-xt}}_{\text{continue et dérivable sur } \mathbb{R}\times\mathbb{R}} dt$$

Soit $a > 0$

$$\left| \frac{\sin(t)e^{-xt}}{t} \right| \leq \left| \frac{e^{-xt}}{t} \right| \leq \left| \frac{e^{-at}}{t} \right| \leq \underbrace{\frac{1}{e^{at}t}}_{\text{série géométrique}} = \frac{1}{e^{at}} \frac{1}{t}$$

Donc $\int_0^{+\infty} \frac{1}{e^{at}t} dt$ cv donc F continue sur $]a, +\infty[$