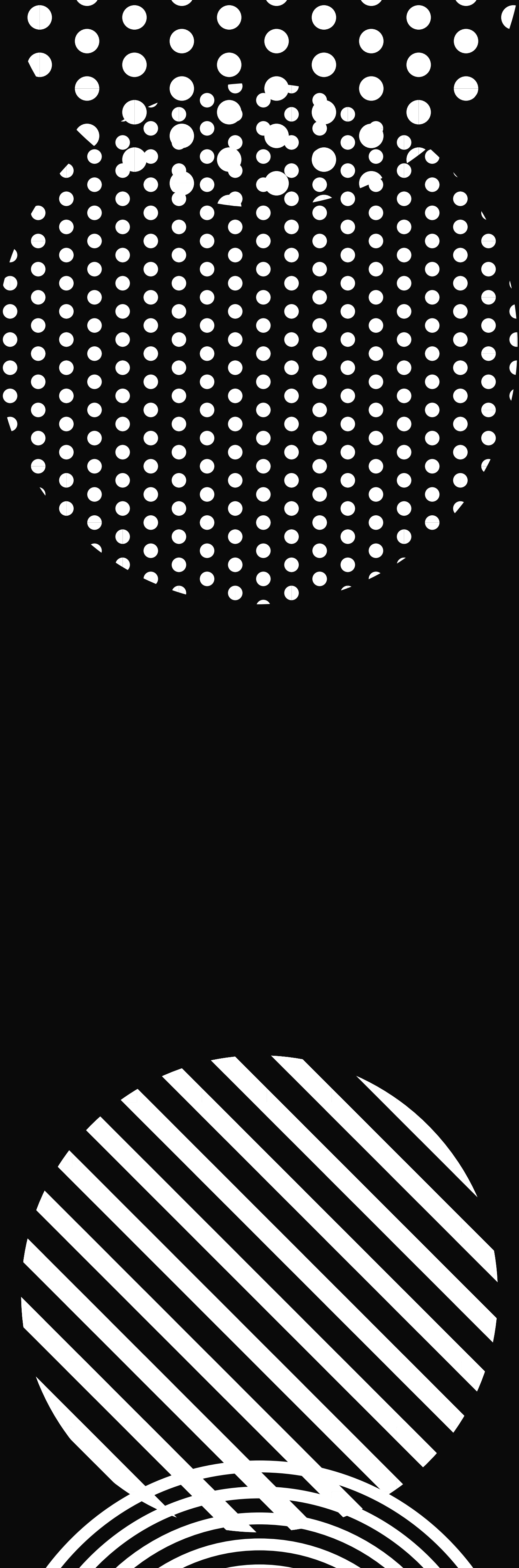
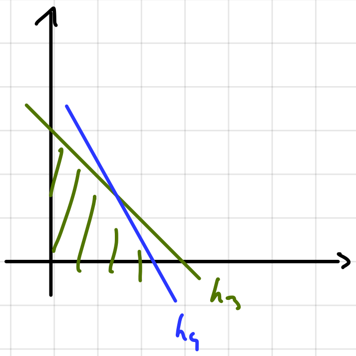


Exercice 3.1

$f(x,y) = 16000x + 10000y$

$h_1(x,y) = -x$
$h_2(x,y) = -y$
caoutchouc : $h_3(x,y) = x+y-400$
$h_4(x,y) = 2x + y - 600$

par le théorème de la solution sommet : la solution est sur un sommet.



On a $C = \{(x,y) : h_1(x,y) \leq 0,\ h_2(x,y) \leq 0,\ h_3(x,y) \leq 0,\ h_4 \leq 0\}$ est compact donc f admet un max sur C.

$$\begin{cases} x + y - 400 = 0 \\ 2x + y - 600 = 0 \end{cases} \Leftrightarrow \begin{cases} y = x = 200 \end{cases}$$

La solution est $x = y = 200$
$f(200,200) = 5\ 200\ 000$ €

Testons les autres sommets (si on n'ignore pas h_1, h_2)

$$\begin{cases} h_1(x,y) = 0 \\ h_2(x,y) = 0 \end{cases} \Leftrightarrow \begin{cases} x = y = 0 \end{cases}$$

$f(x,y) = 0$

$$\begin{cases} h_1(x,y) = 0 \\ h_4(x,y) = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 600 \end{cases}$$ impossible car $h_3(0,600) = 200 > 0$

$$\begin{cases} h_1(x,y) = 0 \\ h_3(x,y) = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 400 \end{cases}$$ $f(0,400) = 4\ 000\ 000$ €

$$\begin{cases} h_2(x,y) = 0 \\ h_4(x,y) = 0 \end{cases} \Leftrightarrow$$ $y = 0$ $x = 300$
$f(300,0) = 4\ 800\ 000$ €

$$\begin{cases} h_2(x,y) = 0 \\ h_3(x,y) = 0 \end{cases} \Leftrightarrow \begin{cases} y = 0 \\ x = 400 \end{cases}$$ impossible car $h_4(400,0) = 200 > 0$.

Donc le max est $5\ 200\ 000$ € atteint en $x = 200$ $y = 200$.

$\mathcal{L} = -f(x,y) - \lambda_1 h_1(x,y) - \lambda_2 h_2(x,y) - \lambda_3 h_3(x,y) - \lambda_4 h_4(x,y)$

$= +16000x + 10000y + \cancel{\lambda_1 x} + \cancel{\lambda_2 y} - \lambda_3(x+y-400) - \lambda_4(2x+y-600)$

$$\frac{\partial \mathcal{L}}{\partial x} = -16000 + \lambda_1 - \lambda_3 - 2\lambda_4 = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = -10000 + \lambda_2 - \lambda_3 - \lambda_4 = 0$$

$$\lambda_1 h_1 = -\lambda_1 x = 0$$

$$\lambda_2 h_2 = -\lambda_2 y = 0$$

$$\lambda_3 h_3 = \lambda_3 (x + y - 400) = 0$$

$$\lambda_4 h_4 = \lambda_4 (2x + y - 600) = 0$$

4. (a) $f_2(u,v) = 400 \cdot u + 600 \cdot v$

$h(u,v): 400 \cdot u + 600 \cdot v - 5200000 \leq 0$

$$\begin{cases} u + v \geq 10000 \\ u + 2v \geq 16000 \\ u \geq 0 \\ v \geq 0 \end{cases}$$

$$\mathcal{L}^* = 400u + 600v - \mu_1(-u - v + 10000) - \mu_2(-u - 2v + 16000) - \mu_3(-u) - \mu_4(-v)$$

$$= \overset{\lambda_3}{400u} + \overset{\lambda_4}{600v} - \overset{y}{\mu_1}(-u - v + 10000) - \overset{x}{\mu_2}(-u - 2v + 16000) + \mu_3(u) + \mu_4(v)$$

$$(x,y) = 400\lambda_3 + 600\lambda_4 - y(-10000 - \lambda_2 + \lambda_3 + \lambda_4) - x(-16000 - \lambda_1 + \lambda_3 + 2\lambda_4)$$

$$= 400\lambda_3 + 600\lambda_4 + y(-10000 - \lambda_3 - \lambda_4) + \cancel{y\lambda_2}$$

$$+ x(-16000 - \lambda_3 - 2\lambda_4) + \cancel{x\lambda_1}$$

primal $\begin{cases} \lambda_3 - 2\lambda_4 = -16000 \\ -\lambda_3 - \lambda_4 = -10000 \end{cases}$ donc $\begin{aligned} \lambda_3 &= \left(\frac{52}{3} - 16\right)10^3 = u \\ \lambda_4 &= \left(\frac{26}{3}\right)10^3 = v \end{aligned}$ sol de problem dual.