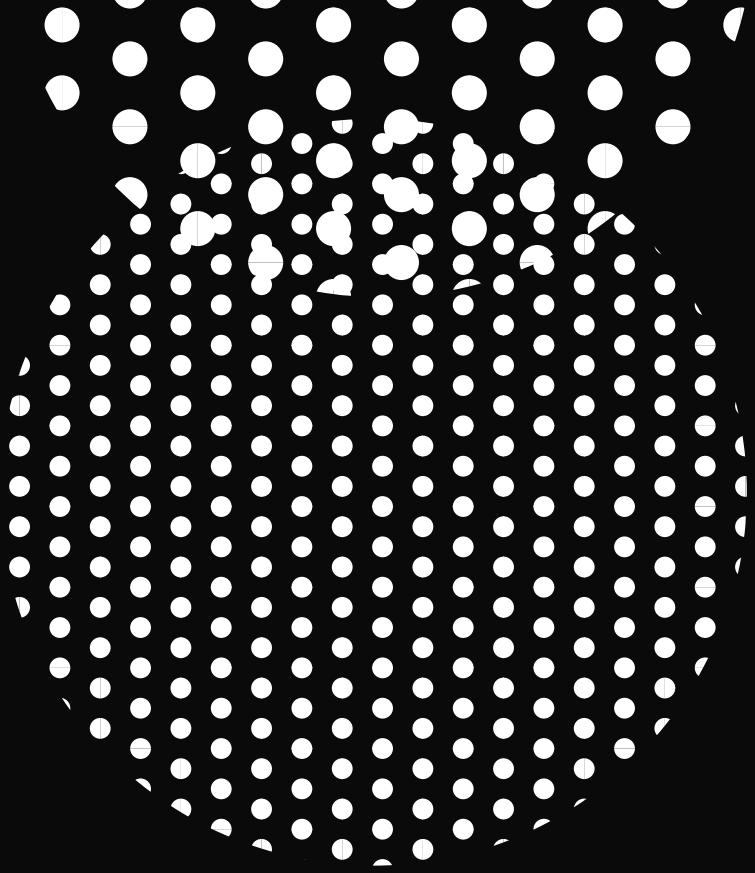

## Exercice 6.2

(6 |

$$\mathcal{I}(A \Leftrightarrow B) = (\mathcal{I}(A) \cap \mathcal{I}(B)) \cup (\bar{\mathcal{I}}(A) \cap \bar{\mathcal{I}}(B))$$

$$|\bar{\mathcal{I}}(A) \cap \bar{\mathcal{I}}(B)| = 2^n - |\mathcal{I}(A) \cup \mathcal{I}(B)|$$

$$= 2^n - |\mathcal{I}(A)| - |\mathcal{I}(B)| + |\mathcal{I}(A) \cap \mathcal{I}(B)|$$

$$|\mathcal{I}(A \Leftrightarrow B)| = 2^n - |\mathcal{I}(A)| - |\mathcal{I}(B)| + 2\,|\mathcal{I}(A) \cap \mathcal{I}(B)|$$

c)

## Exercice 7.1

1. $\vdash (p \wedge q), (\neg p \wedge \neg q)$

$(p \wedge q) \vee (\neg p \wedge \neg q)$ ← forme associée

hyp $\dfrac{}{p \vdash p}$ — aucune règle ne s'applique

$\wedge$ $\dfrac{}{p \vdash q}$

$\neg d$ $\dfrac{p \vdash p \wedge q}{\vdash (p \wedge q), \neg p}$

$$\frac{\vdash (p \wedge q), \neg p \qquad \vdash (p \wedge q), \neg q}{\vdash (p \wedge q), (\neg p \wedge \neg q)} \wedge d$$

2. $p, \bot \vdash$ forme associée $(p \wedge \bot) \Rightarrow \bot$

$$\frac{}{p, \bot \vdash} \bot g$$

$$\frac{\dfrac{p \vdash}{p \vee \neg p \vdash}}{\vdash \neg(p \vee \neg p)} \neg d \wedge d$$

3. $\top \vdash$ forme associée $\top \Rightarrow \bot$

Jamais tout le temps $\Rightarrow$ formule racine insatisfiable.

$$\dfrac{\vdash}{\top \vdash}\ \top g$$

4. $\vdash$

## Exercice 7.2

1.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p \vdash p,z}\,hyp}{\neg p, p \vdash p,z}\,\neg g \quad \dfrac{}{q,p \vdash p,z}\,hyp}{\neg p \vee q, p \vdash p,z}\,\vee g \quad \dfrac{\dfrac{\dfrac{}{p \vdash q,z,p}\,hyp}{\neg p, p \vdash q,z}\,\neg g \quad \dfrac{}{z,p \vdash q,z}\,hyp}{\neg p \vee q, p \vdash q,z}\,\vee g}{\neg p \vee q, p \vdash p \wedge q, z}\,\wedge d \quad \dfrac{\dfrac{}{z, p, (\neg p \vee q) \vdash z}\,hyp}{z, p, (\neg p \vee q) \vdash (p \Rightarrow z)}\,\Rightarrow g}{\vdash p \wedge q, (\neg p \vee q) \Rightarrow (p \Rightarrow z)}\,\Rightarrow g}{(p \wedge q) \Rightarrow z \ \vdash\ (\neg p \vee q) \Rightarrow (p \Rightarrow z)}\,\Rightarrow d}{\vdash \big((p \wedge q) \Rightarrow z\big) \Rightarrow \big((\neg p \vee q) \Rightarrow (p \Rightarrow z)\big)}\,\Rightarrow d$$

2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash p}\quad \dfrac{}{\vdash p, q}}{\vdash p, p \wedge q}\,\wedge d \quad \dfrac{}{q \vdash p}}{\neg p \vdash p, p \wedge q}\,\neg g \quad \dfrac{}{q \vdash p, q}\,hyp}{p \vee q \vdash p, p \wedge q}\,\vee g \quad \dfrac{}{z, \neg p \vee q \vdash p}\,hyp}{(p \wedge q) \Rightarrow z, \neg p \vee q \vdash p}\,\Rightarrow g \quad \dfrac{}{b \vee d, d \vdash b \vee d}\,ax}{(p \wedge q) \Rightarrow z, (\neg p \vee q) \vdash b \vee d}\,\vee d}{((p \wedge q) \Rightarrow z) \vee (\neg p \vee q) \vdash b \vee d}\,\vee g}{\vdash \big(((p \wedge q) \Rightarrow z) \vee (\neg p \vee q)\big) \Rightarrow (b \vee d)}\,\Rightarrow d$$

## Esercizio 7.3

1.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{R(x,3),\forall y,R(x,y)\vdash R(x,3),\exists t,R(t,3)}\,hyp}{\forall y,R(x,y)\vdash \exists t,R(t,3)}\,\forall\ y\in 3}{\forall y,R(x,y)\vdash \exists t,R(t,3)}\,\exists\ t\leftarrow x}{\forall y,R(x,y)\vdash \forall 3,\exists t,R(t,3)}\,3}{\exists x,\forall y,R(x,y)\vdash \forall 3,\exists t,R(t,3)}\,x$$

Bonus

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B(y),B(a)\vdash B(y),\exists x\dots}\,hyp}{B(a)\vdash B(y),(B(y)\Rightarrow\forall 3\,B(3))\,\exists x\dots}\,\exists\ x\leftarrow y}{B(a)\vdash B(y)\qquad(\exists x,(B(x)\Rightarrow\forall y,B(y)))}\,\forall\ new\ y}{B(a)\vdash\forall y,B(y),\exists x,(B(x)\Rightarrow\forall y,B(y))}$$

$$\dfrac{\vdash B(a)\Rightarrow\forall y,B(y),\exists x,(B(x)\Rightarrow\forall y,B(y))}{\vdash\exists x,(B(x)\Rightarrow\forall y\,B(y))}\,\exists\ x\leftarrow a$$

2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A,R(x',3),R(x,a)\vdash B,R(b,3)}{A,(\exists x,R(x,3)),R(x,a)\vdash B,R(b,3)}\,\forall\ y\quad y\leftarrow 3}{A,R(x,a)\vdash B,R(b,3)}}{A,R(x,a)\vdash B,\forall 3\,R(b,3)}}{A,R(x,a)\vdash B}}{A,\exists x,R(x,a)\vdash B}}{\underbrace{\forall y,\exists x,R(x,y)}_{A}\vdash\underbrace{\exists t,\forall 3,R(t,3)}_{B}}\,a$$