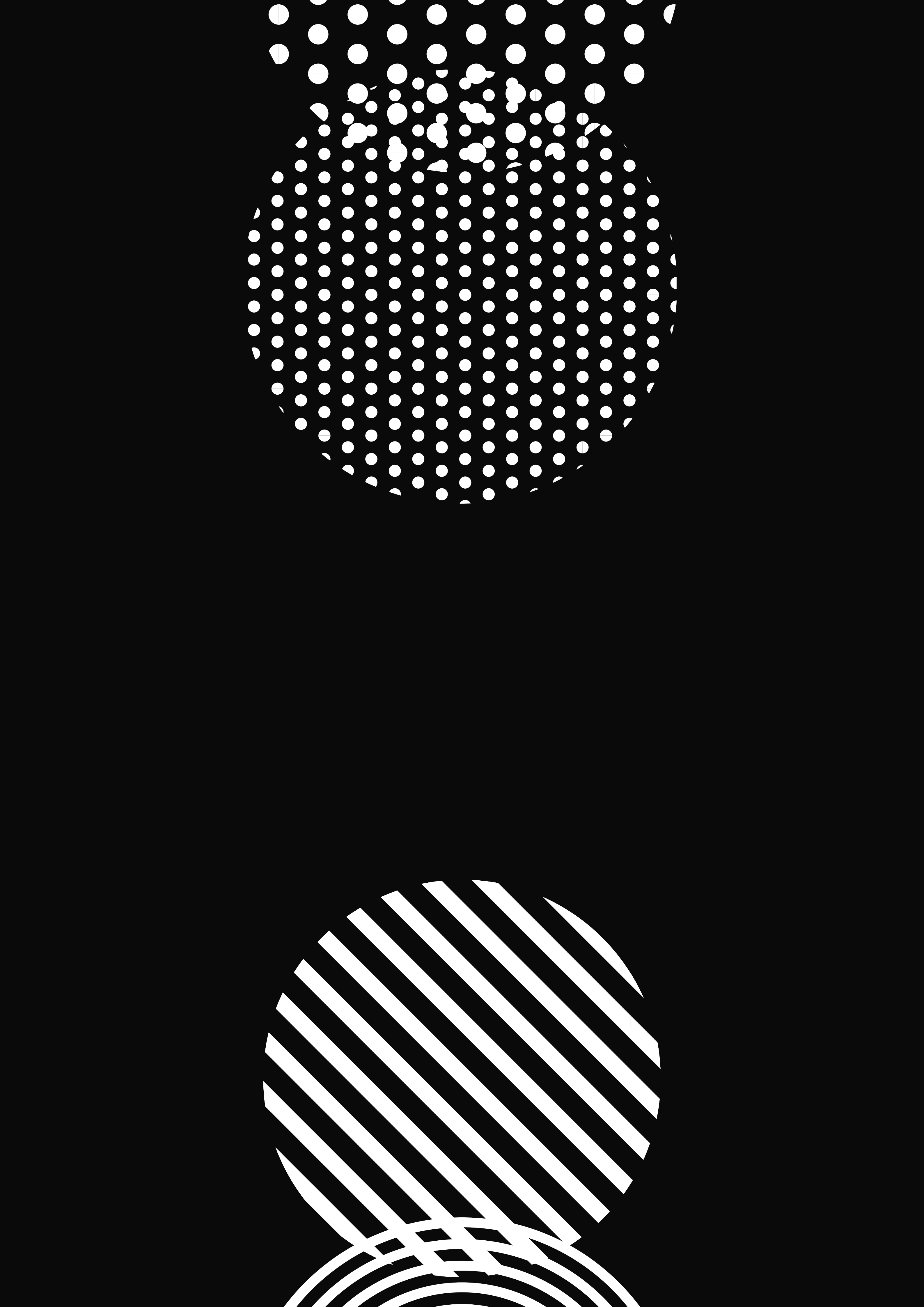

Exercice 10.1

$$p_\lambda(x_1,\dots,x_N) = \prod_{i=1}^{N} p_\lambda(x_i) = \prod \lambda e^{-\lambda x_i} = \lambda^N e^{-\lambda(\sum x_i)}$$

$$\log(p_\lambda(x_1,\dots,x_N)) = \log(\lambda^N e^{-\lambda \sum x_i}) = \log(\lambda^N) + \log(e^{-\lambda \sum x_i})$$

$$= N\log(\lambda) - \lambda \sum_{i=1}^{N} x_i$$

$$\frac{d}{d\lambda}\log(p_\lambda(x_1,\dots,x_N)) = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i$$

$$\frac{d}{d\lambda}\log(p_\lambda(x_1,\dots,x_N)) = 0 = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i$$

$$\Leftrightarrow \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{\lambda} \quad \Leftrightarrow \lambda = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} x_i}$$

Comme log est concave donc trouver un extremum local suffit pour savoir que c'est un maximum global.