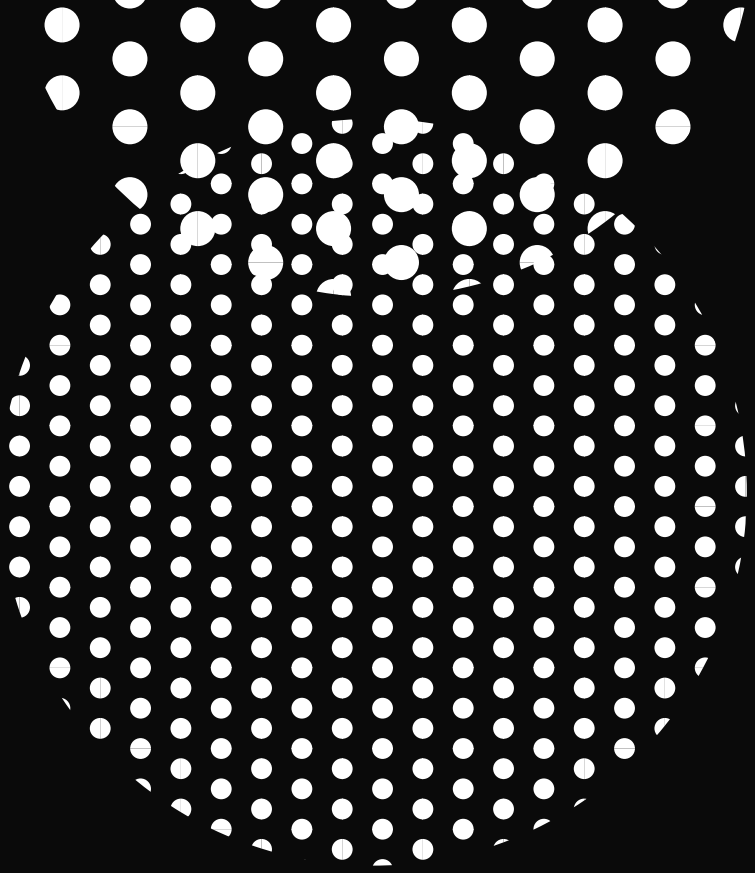


Exercice 1

1.

$$\dfrac{\dfrac{\dfrac{}{t:int[] \vdash t:int[]}\,Var \quad \dfrac{}{t:int[] \vdash 1:int}\,Var}{t:int[] \vdash t[1]:int}\,Get \quad \dfrac{}{t:int[] \vdash 2:int}\,Var}{\Gamma \vdash t[1]+2:int}\,Get$$

2.

$$\dfrac{\dfrac{}{t:int \vdash t:int[]}\,Var \quad \dfrac{\dfrac{}{t:int[] \vdash t:int[]}\,Var \quad t: \vdash 3:int}{t:int[] \vdash t[3]:int}\,Get}{\Gamma \vdash t[t[3]]:S}\,Get$$

donc $\Gamma = \{t:int[]\}$

3.

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash t:S[][]}\,Var \quad \Gamma \vdash 3:int}{\Gamma \vdash t[3]:S[]} \quad \Gamma \vdash 4:int}{\Gamma \vdash t[3][4]:S}\,Get$$

donc $\Gamma = \{t: S[][]\}$

4.

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash t:S[][]}\,Var \quad \Gamma \vdash 3:int}{\Gamma \vdash t[3]:S[]} \quad \dfrac{\dfrac{}{\Gamma \vdash t:int[]}\,Var \quad \Gamma \vdash 4:int}{\Gamma \vdash t[4]:int}\,Get}{\Gamma \vdash t[3][t[4]]:S}\,Get$$

impossible

5.

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash t:S[]}\,Var \quad \Gamma \vdash 1:int}{\Gamma \vdash t[1]:S}\,Get \quad \dfrac{\Gamma \vdash t:S[] \quad \Gamma \vdash 3:int}{\Gamma \vdash t[3]:S}\,Get}{\Gamma \vdash [t[1], t[3]]:S[]}\,Arr$$

6.

$$\dfrac{\dfrac{}{\Gamma \vdash t : \mathcal{T}}\,Var \qquad \dfrac{\dfrac{}{\Gamma \vdash t : \mathcal{T}[]}\,Var \qquad \Gamma \vdash 0 : int}{\Gamma \vdash t[0] : \mathcal{T}}\,Get}{\Gamma \vdash [t, t[0]] : \mathcal{T}[]}\,Arr$$

mal typé.

Exercice 2

$e : \mathcal{T}_1 \times \mathcal{T}_2$

$fst(e) : \mathcal{T}_1$
$snd(e) : \mathcal{T}_2$

$$\dfrac{\Gamma \vdash e_1 : \mathcal{T}_1 \qquad \Gamma \vdash e_2 : \mathcal{T}_2}{\Gamma \vdash (e_1, e_2) : \mathcal{T}_1 \times \mathcal{T}_2}$$

$$\dfrac{\Gamma \vdash e : \mathcal{T}_1 \times \mathcal{T}_2}{\Gamma \vdash fst(e) : \mathcal{T}_1}$$

$$\dfrac{\Gamma \vdash e : \mathcal{T}_1 \times \mathcal{T}_2}{\Gamma \vdash snd(e) : \mathcal{T}_2}$$

Exercice 3

$$\dfrac{}{\Gamma \vdash true : Bool} \qquad \dfrac{}{\Gamma \vdash false : Bool}$$

$$\dfrac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : Bool}{\Gamma \vdash e_1 < e_2 : Bool}$$

$$\dfrac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : Bool}{\Gamma \vdash e_1 = e_2 : Bool}$$

$$\dfrac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : Bool}{\Gamma \vdash e_1 \&\& e_2 : Bool}$$

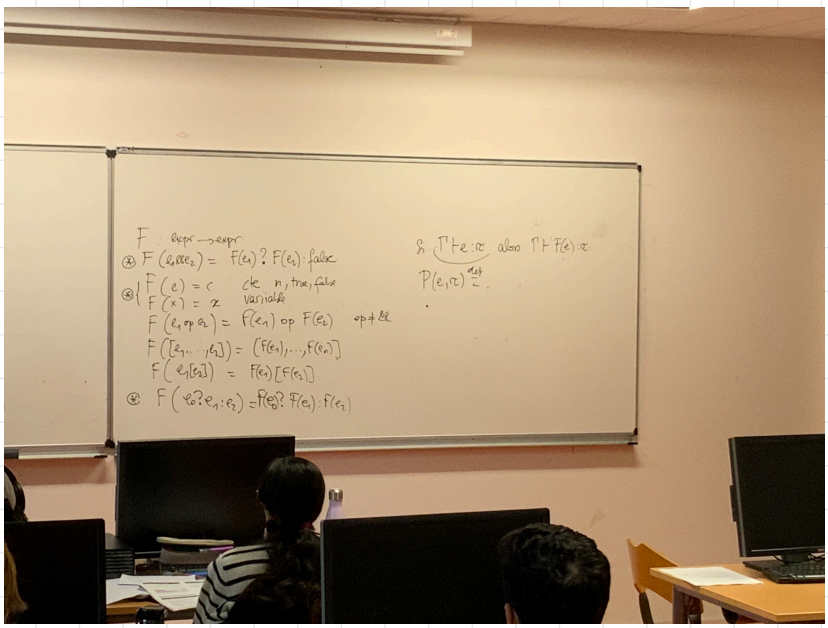
$$\frac{\Gamma \vdash e_0 : \text{bool} \qquad \Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau'}{\Gamma \vdash e_0 \,?\, e_1 : e_2 \; : \; \tau''}$$

Exercice 4

$F(\text{true}) = \text{true}$
$F(\text{false}) = \text{false}$

$$F(e_1 \;\&\&\; e_2) = \begin{cases} \text{true} & \text{if } F(e_1) \text{ and } F(e_2) \\ \text{false} & \text{otherwise} \end{cases}$$

$F(e) = e$ si e ne contient pas $\&\&$



Si $\Gamma \vdash e : \tau$ alors $\Gamma \vdash F(e) : \tau$

$P(e, \tau) \overset{\text{déf}}{=} \Gamma \vdash F(e) : \tau$

$$\frac{}{\Gamma \vdash n : \text{int}} \text{ axiom}$$

$$\frac{\Gamma \vdash F(n) : \text{int}}{= \Gamma \text{ HR}} \quad \begin{cases} P(e_1, \tau) \text{ vrai} \\ P(e_2, \tau) \text{ vrai} \end{cases}$$

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash F(e_1 \text{ op } e_2) : \tau'} \text{ op}$$

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash F(e_1) \text{ op } F(e_2) : \tau'} \text{ op}$$

$$\frac{\Gamma \vdash e_1 : bool \quad \Gamma \vdash e_2 : bool}{\Gamma \vdash e_1 \,\&\&\, e_2 : bool}$$

Si $P(e_1, bool)$ et $P(e_2, bool)$ alors $P(e_1 \,\&\&\, e_2, bool)$

$$\frac{\Gamma \vdash F(e_1) : bool \overset{HR}{\Rightarrow} \Gamma \vdash F(e_1) : bool \qquad \Gamma \vdash F(e_2) : bool \overset{HR}{\Rightarrow} \Gamma \vdash F(e_2) : bool \qquad \Gamma \vdash false : bool}{\Gamma \vdash F(e_1) \,?\, F(e_2) : false : bool}$$

Exercice 5

Si $\Gamma, x : \tau' \vdash e : \tau$ et $\Gamma : e' : \tau'$ alors $\Gamma \vdash e^{\{x \to e'\}} : \tau$

- recurrence sur la dérivation de $\Gamma, x : \tau' \vdash e : \tau$