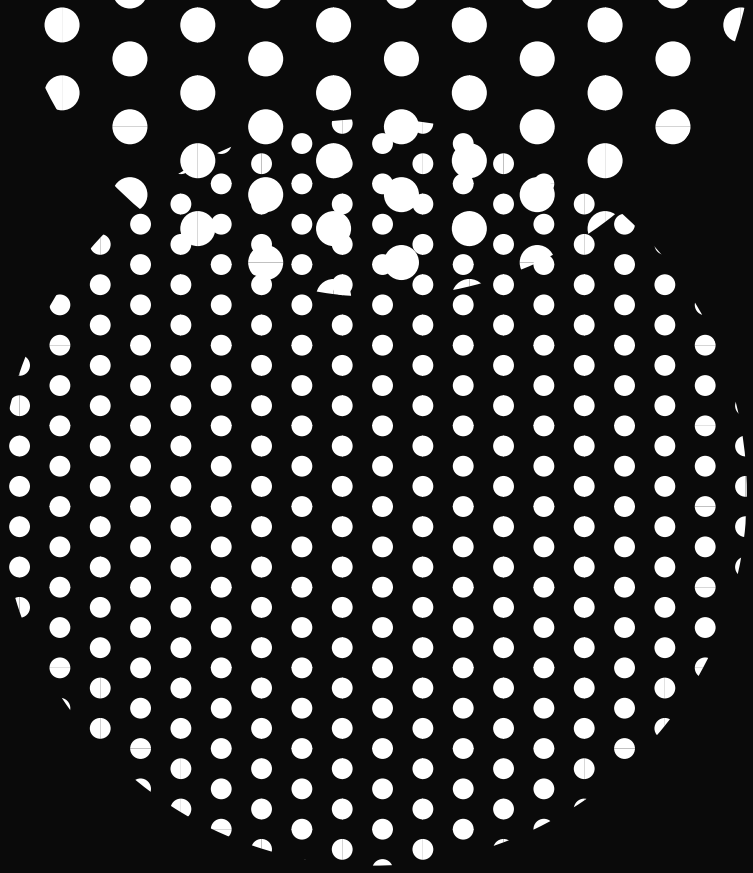


Exercice 1

1.

$N(T, N(T, N(\bot, E, E), N(\bot, E, E)),$

$N(\bot, N(\bot, E, E), N(T, N(T, E, E), N(\bot, E, E))$

2. $size(E) = 0$

$size(N(b, t_0, t_1)) = size(t_0) + size(t_1) + (1 \text{ si } b = T \text{ et } 0 \text{ si } b = \bot)$

3.

$union(E, E) = \{\}$

$union(t_0, E) = t_0$

$union(E, t_0) = t_0$

$union(N(b_0, a_0, b_0), N(b_1, a_1, b_1)) = N(b_0 \| b_1, union(a_0, a_1), union(b_0, b_1))$

4.

$size(union(t_0, E)) = size(union(E, t_0)) = size(t_0)$

donc $size(t_0) \leq size(union(t_0, E)) \leq size(t_0) + size(E)$

$size(union(E, t_0))$ $= size(t_0)$

Supposons que $size(t_0) \leq size(union(t_0, t_1)) \leq size(t_0) + size(t_1)$
sous toutes les combinaison

$size(union(N(b_0, t_0, t_1), N(b', t_0', t_1')))$

$= size(N(b_0 \| b', union(t_0, t_0'), union(t_1, t_1')))$

$= size(union(t_0, t_0')) + size(union(t_1, t_1')) + b_0 \| b'$

$\leq size(t_0) + size(t_0') + size(t_1) + size(t_1') + b_0 \| b'$

$\leq (size(t_0) + size(t_1) + b_0) + (size(t_0') + size(t_1') + b')$

$= size(N(b_0, t_0, t_1), N(b', t_0', t_1'))$

$size(N(b_0, t_0, t_1)) = size(t_0) + size(t_1) + b_0$

$\leq size(union(N(b_0, t_0, t_1), N(b', t_0', t_1')))$

$=$

$size(union(t_0, t_0')) + size(union(t_1, t_1')) + b_0 \| b'$

car $size(t_0) \leq size(union(t_0, t_0'))$

$size(t_1) \leq size(t_1, t_1')$

$b_0 \leq b_0 \| b'$

D'où conclusion

5. Soient l, t_1, t_2, t_3, t_4 t_4

$\forall i \in J \quad in(l, t_i) \Rightarrow \forall j \in J \setminus \{i\} \quad in(l, union(t_i, t_j))$

$in(l, N(b, t_1, t_2))$

$union(N(b, t_1, t_2), N(b', t_3, t_4)) = N(b \| b', union(t_1, t_3), union(t_2, t_4))$

$in(ol, N(b, t_1, t_2))$ donc $in(l, t_1)$ cela donne $in(l, union(t_1, t_3))$
$in(1l, N(b, t_1, t_2))$ donc $in(l, t_2)$ cela donne $in(l, union(t_2, t_4))$

donc soit $in(ol, N(b, t_1, t_2))$ vrai

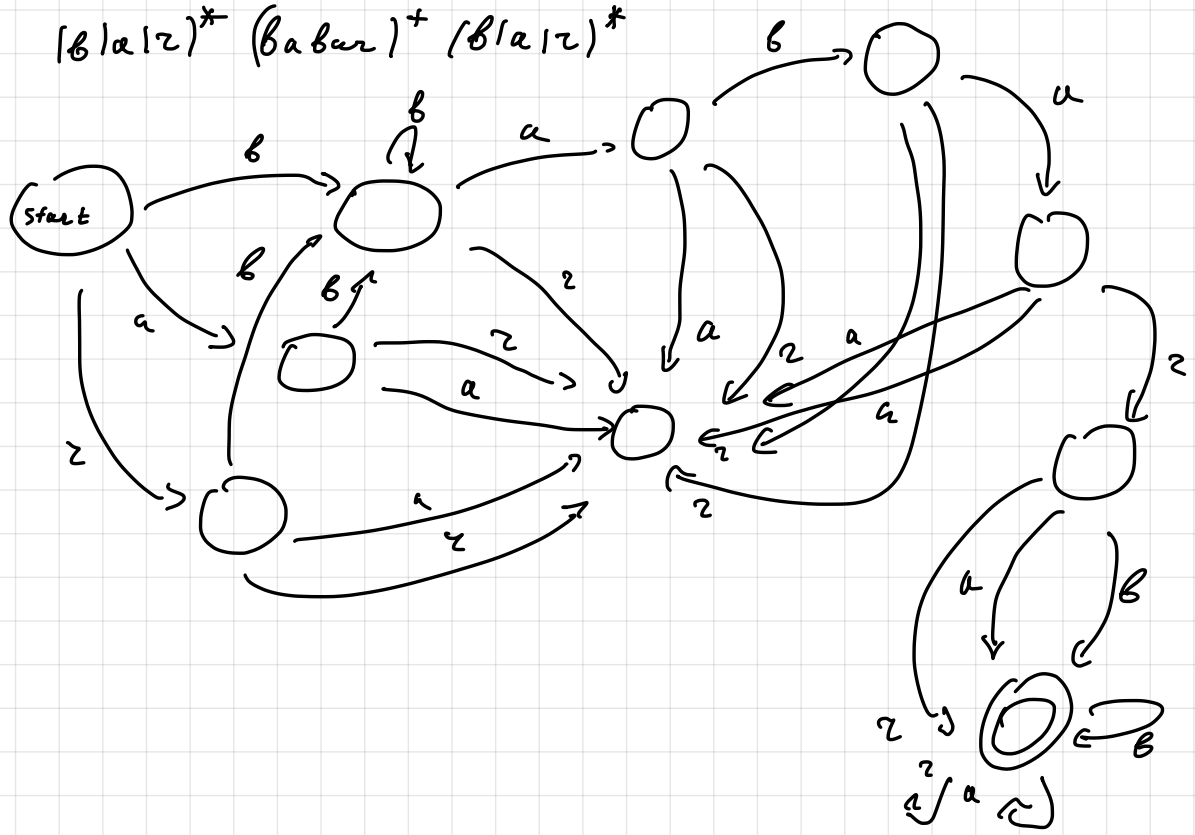
$in(ol, union(N(b, t_1, t_2), N(b', t_3, t_4)))$

$= in(ol, N(b \| b', union(t_1, t_3), union(t_2, t_4))) = in(l, union(t_1, t_3))$ vrai par la symmétrique pour l'autre

Exercice 2

1. $(b|a|\tau)^* (b a b a \tau)^+ (b|a|\tau)^*$

2.

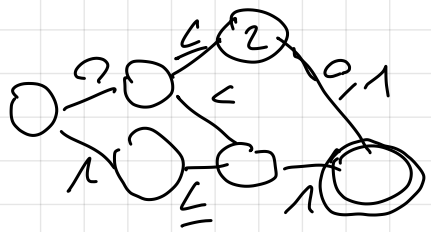


Exercice 3

1. $0 \leq 0$
   $0 \leq 1$
   $0 < 1$
   $1 \leq 1$

$$\boxed{\begin{array}{c} (0 \leq (0 \mid 1)) \mid (0 < 1) \mid 1 \leq 1 \\ \hline (0 \mid (\leq \mid <) \mid 1) \mid (0 \leq 0) \mid 1 \leq 1 \end{array}}$$

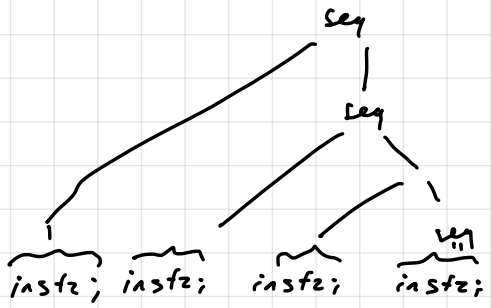
2.



Exercice 4

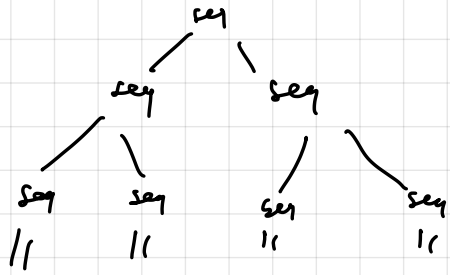
1. Ge, Gc, Gb

Ge:



Gc:

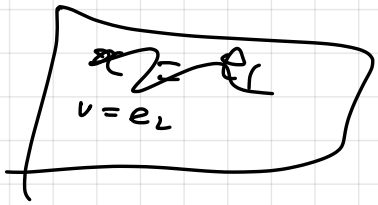
G_B:



2. G_a, G_B

3. G_e, G_c, G_B

4. G_a: instr

   G_d: ;instr

e_1 e_2