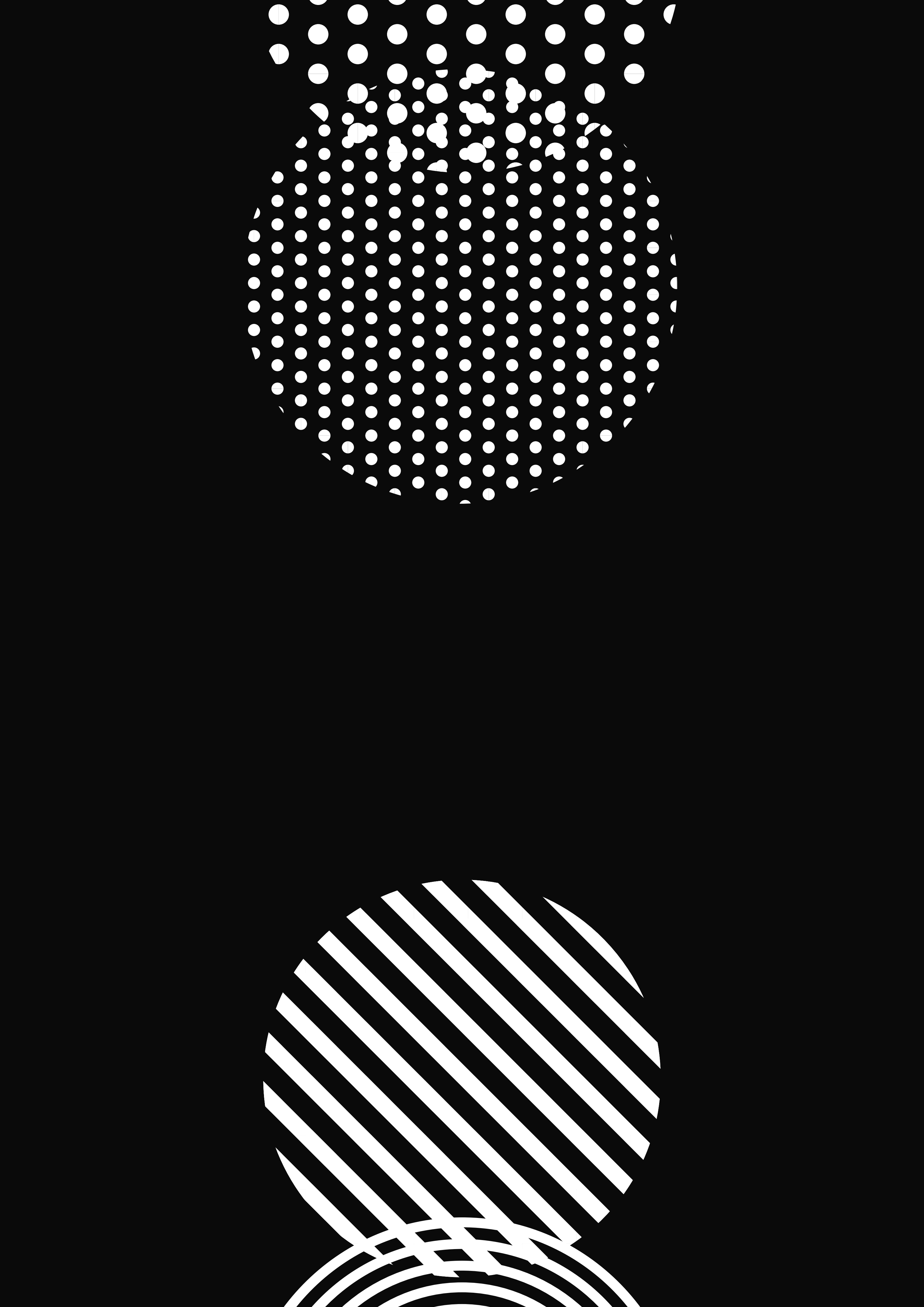

Exercice 1

$$x = c\begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{aligned} Ax - b &= cA\begin{pmatrix}1\\0\end{pmatrix} - b \\ &= c\begin{pmatrix}1\\3\end{pmatrix} - b \\ &= c\begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} \end{aligned}$$

$\min \|Ax - b\|$ $\qquad\qquad$ $x = c\begin{pmatrix}1\\0\end{pmatrix}$

$$\left\langle c\begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}, c\begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} \right\rangle$$

$$Ax = c\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}c\\3c\end{pmatrix}$$

$$\underbrace{Ax - b}_{\begin{pmatrix}1\\0\end{pmatrix}} = \begin{pmatrix}c-1\\3c\end{pmatrix} \perp \mathcal{X}$$

$$\left\langle \begin{pmatrix}c-1\\3c\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix} \right\rangle = 0$$

$$c - 1 = 0$$

$$c = 1$$

Exercice 2

$$\min_{x \in \mathcal{X}} \underbrace{\frac{1}{2}\langle x, Ax\rangle - \langle b, x\rangle}_{f(x)}$$

$f_{|\mathcal{X}}$ a un unique min

$$f(x) = \frac{1}{2}\langle x, Ax\rangle - \langle b, x\rangle \geq \frac{1}{2}\lambda_1(A)\|x\| - \|b\|\,\|x\| \xrightarrow[\|x\| \to +\infty]{} +\infty$$

- $f(x) \xrightarrow[\substack{\|x\| \to +\infty \\ x \in \mathcal{X}}]{} +\infty$ et f continue $\Rightarrow \exists$ minimum

Montrer que $\nabla f_{|\mathcal{X}}(x) = 0$ admet une unique solution

$$\nabla f_{|\mathcal{X}} = \pi_{\mathcal{X}} \nabla f$$

$\pi_{\mathcal{X}}$

$\pi_{\mathcal{X}} \nabla f(x) = 0$ $\quad$ unique solution

$$\underbrace{(\pi_{\mathcal{X}} A \pi_{\mathcal{X}})}_{\text{injectif?}} x = \pi_{\mathcal{X}} b$$

$\Pi_X A \Pi_X = 0 \Leftrightarrow \langle \Pi_X A \Pi_X x, x \rangle = 0$

$\Leftrightarrow \langle A \Pi_X x, \Pi_X x \rangle = 0$

$\Rightarrow x = 0$ comme A inject.

Optimalité

$x_* \in \mathbb{R}^N$ solution du pb global
$x'_* \in \mathcal{X}$ solution du pb restreint

$\|x_* - x'_*\| = \min_{y \in \mathcal{X}} \|x_* - y\|$

$\Leftrightarrow \quad x'_* = \Pi_X$,