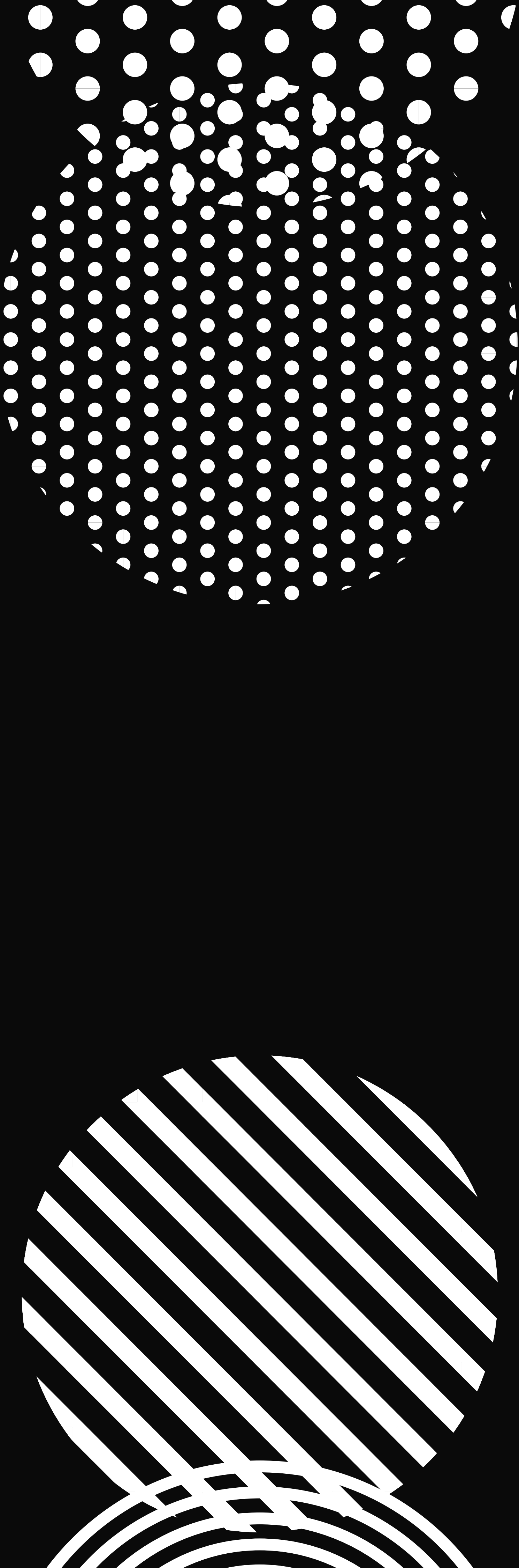

1, 3, 5, 10

Exercice 1

$A = \Big($

M3

$\begin{pmatrix} x \\ y \end{pmatrix}^T A \begin{pmatrix} x \\ y \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 \\ 0.1 & 2 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 1-\lambda & 0 \\ 0.1 & 2-\lambda \end{pmatrix}$

$\det(A - \lambda I) = (1-\lambda)(2-\lambda)$

Donc 1 et 2 sont des valeurs propres

Donc deux valeurs propres sont strictement positives donc la matrice est définie positive.

M1: $|xy| \leq \frac{1}{2}x^2 + \frac{1}{2}y^2$

$q(x,y) \geq x^2 - 0.1|xy| + y^2$

$\geq 0.95x^2 + 1.95y^2 \geq 0 \quad \forall (x,y) \in \mathbb{R}^2$

donc définie positive

M2: $\forall (x,y) \in \mathbb{R}^2$

$q(x,y) = (x + 0.05y)^2 + 2y^2 - 0.0025y^2$
$= (x + 0.05y)^2 + 1.9975y^2 \geq 0$

$q(x,y) = 0 \Leftrightarrow x + 0.05y = 0$ et $y = 0 \Leftrightarrow x = y = 0$

Exercice 3

$$c_{i,j} = \sum_{k=1}^{N} a_{i,k} \, x_{k,j}$$

$$z_{i,j} = \sum_{l=1}^{N} x^*_{i,l} \sum_{k=1}^{N} a_{l,k} \, x_{k,j}$$

Donc $\forall i,j > n \quad x_{i,j} = 0$ et

$$(X^* A X)_{i,j} = \sum_{k,j \geq 0}^{n} \delta_{i,l} \, a_{l,k} \, \delta_{k,j}$$

$$X = \begin{pmatrix} \mathcal{I}_n \\ 0_{N-n} \end{pmatrix}$$

$X = (v_1, \dots, v_N)$

On veut $\forall A$, $\forall i,j \leq n$

$$\langle v_i, A v_j \rangle = \langle e_i, A e_j \rangle$$

$\forall i,j \leq n$

$$\langle v_i, \mathcal{I}_N, v_j \rangle = \langle e_i, e_j \rangle \iff \langle v_i - e_i, A(v_j - e_j) \rangle = 0$$

$$\langle v_i, v_j \rangle = \langle e_i, e_j \rangle$$

$$\langle v_i, A v_j \rangle = \langle e_i, A e_j \rangle$$

$$\langle e_i, (X^* A X) e_j \rangle = \langle X e_i, A X e_j \rangle$$

On veut

$$\langle X e_i, A X e_i \rangle = \langle e_i, A e_j \rangle \quad \forall i,j \leq n$$

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} (V_1, V_2)$$

Exercice 5

$$v''(x_n) = \frac{2v(x_n) - v(x_{n-1}) - v(x_{n+1})}{\left(\frac{1}{N^2}\right)}$$

$$x_n = -L + \frac{n}{N} 2L$$

$$\Delta_N = \begin{pmatrix} 2 & -1 & -1 & \ddots & & \\ -1 & 2 & \ddots & \ddots & \ddots & \\ & -1 & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots & -1 \\ & & & \ddots & -1 & 2 \end{pmatrix}$$

$\forall k$

$$\Delta_N \underbrace{\begin{pmatrix} \sin(\pi k \frac{1}{N}) \\ \vdots \\ \sin(\pi k) \end{pmatrix}}_{v_k} = -2N^2 \left(\cos\left(\frac{k\pi}{N}\right) - 1 \right) \begin{pmatrix} \sin(\pi k \frac{1}{N}) \\ \vdots \\ \sin(\pi k) \end{pmatrix}$$

$$(-\Delta_N v_k)_n = N^2 \left(2 \sin\left(\pi k \frac{n}{N}\right) - \sin\left(\pi k \frac{(n-1)}{N}\right) - \sin\left(\pi k \frac{(n+1)}{N}\right) \right)$$

формула залупы

$$= N^2 \left(2 \sin\left(\frac{\pi k n}{N}\right) - 2 \sin\left(\frac{\pi k n}{N}\right) \cos\left(\frac{\pi k}{N}\right) \right)$$

$$= 2 N^2 \sin\left(\frac{\pi k n}{N}\right) \left(1 - 2 \cos\left(\frac{\pi k n}{N}\right) \right)$$

$N \gg k$ $\qquad \sin\left(\frac{\pi k n}{N}\right) \sim \frac{\pi k n}{N}$

$$2 N^2 \left(1 - \cos\left(\frac{\pi k}{N}\right) \right) \sim \frac{\pi^2 k^2}{N^2} \cdot 2 N^2 = 2 \pi^2 k^2$$